Application of Linear Programming in Forestry: Management Planning for a Forest Plantation

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Abstract

Long–term planning for the management of the regulated forests can be a difficult undertaking. The forest planning process consists of four stages, namely, data preparation, generation of management alternatives, selection of management alternatives and management plan formulation. In this paper, the advantage of a linear programming model in forestry is described and a forest plantation of the Forest Industry Organization, a teak plantation, is taken as an example. The model permits specification of the management program in an irregular trend that will maximize the net present value of expected yields while converting the irregular forest to regulated plantations in a finite period. At the end of the planning horizon, the forest plantation age/size class distribution would be such that the forest could produce approximately equal annual or periodic harvests of timber.

Keywords: Linear programming, Forest management planning, Forest plantation.

1. Introduction

Managers of economic enterprises all share the same problem of efficiently allocating the factors of production–labor, capital, and natural resources. In the forest enterprise, this allocation is largely controlled by the forest manager’s decisions on where, when, and how much to cut. Several complexities make these decisions particularly difficult. Primary among these are (1) the long–term nature of the production process which introduces a significant degree of uncertainty concerning pertinent future economic and biological conditions, and (2) the essentially unlimited number of possible cutting strategies (Ware and Clutter, 1971).

Achieving the goal of sustainable forest management (SFM) calls for the formulation of a forest regulation model that could provide for the continuous flow of timber products of the desired quantity and quality. Such a forest regulation model should be able to produce yields for an annual or periodic crop of about equal volume, size, and quality. The ultimate goal of forest management is the maximization of the utility of the forest to the owner. If the owner is profit oriented, this utility can usually be measured in terms of quantities such as net present value, return on investment, or cost of specified wood flow.

Linear programming (LP) is one of the most widely used methods for solving long-term forest management problems (Ware and Clutter 1971; Dargavel and Bethune,
1972; Johnson and Scheurman 1977; Leuschner, 1990). One reason for this is the computational efficiency of LP. Another is the versatility by which a constraint matrix can be formed out of treatment schedules generated by practically any stand projection model (Ohman and Eriksson, 2002). The application of linear programming for plantation management is directed to yield control; that is to scheduling stand for planting, thinning, and final cutting such that wood production targets are reached. The purpose of this paper is to describe a linear programming model used in the management planning for forest plantation. A teak plantation of the Forest Industry Organization (FIO), Thong Pha Phum Plantation, will be illustrated as a case study.

2. Forest Management Planning

2.1 The Principle

Forests are usually managed to obtain a sustained yield of products, historically timber. A sustained yield requires a constant production level for a particular management intensity. This means the incremental growth of the forest and the harvest must be brought into balance. Thus, the sustained yield is not necessarily what is being produced today, but is the potential yield from a forest, once the management intensity is decided upon and increment and harvest are balanced.

Practical forest planning is not only preference analysis and optimization. Inventory, data management, calculation and computer simulation all play important roles in the process that leads to a forest plan. Forest planning compares alternative forest management plans, which may also be called decision alternatives, aiming at finding a decision that has the best possible consequences. Various models and calculation techniques are used to predict the consequences of decision alternatives. This requires predicting the stand development under alternative management options and calculating the amounts of those products and services, which are important to the owner. The predictions use computer simulation and are based on forest inventory data stored in a database as well as on models and calculation methods (Pukkala, 2002).

Managed forests are typically divided into mutually exclusive and geographically identifiable areas referred to as “cutting units”. Methods for delineating cutting units vary in practice but the important aspect of the concept, as used here, is the anticipation that cutting units will be even-aged homogeneous stands in subsequent rotations. Each year, the forest manager must schedule cutting in some cutting units. These decisions will not, ordinarily, be made without considerable study and long-range planning. Typically, a management plan will be prepared listing the cutting units to be harvested and/or regenerated during each year of a planning period. This “cutting schedule” can list units scheduled for partial cutting and units scheduled for final cutting.

The forest managers believe that the preparation of a cutting schedule is the heart of any timber management operation, for it is primarily through the temporal and spatial scheduling of his harvest operations that the forester controls quantities such as growing stock volumes, growth rates, cash flow, present worth, and return on investment. Admittedly, even the best cutting schedule is of little value if activities such as protection, harvesting, regeneration, and marketing are incompetently handled. Conversely, the most skilled practitioners in these endeavors cannot ameliorate more than mildly the economic and biological handicaps created by an ineptly prepared cutting schedule (Ware and Clutter, 1971)
2.2 Linear Programming as a Planning Tool

In the context of planning, the model is called a planning model. The planning model is filled with data and solved with a suitable procedure. The data that are included in the planning model fall into two categories and come from two sources. First, data on preferences, which is obtained from the decision maker and second, data on forest production, which is based on the combined use of forest inventory data and models, unit prices, etc. (Pukkala, 2002). The application of techniques of varying degrees of complexity and comprehensiveness calls for the use of a range of mathematical and data processing tools. It is impossible to sort these tools into any uniquely ordered hierarchy; each is applied as appropriate to the planning problem and technique of solution adopted. Linear programming is used as a solution technique. The forest-planning problem is therefore formulated as a linear programming model.

The general problem of linear programming is the search for the optimum (minimum or maximum) of a linear function of variables constrained by linear relations (equations or inequalities) called constraints (Kilkki, 1968). A linear program optimizes a linear objective function subject to a set of linear equalities and/or inequalities. The general maximization models is (Leuschner, 1990):

\[ \text{Max } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \]  

Subject to

\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq r_1 \]  
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq r_2 \]  
\[ \vdots \]  
\[ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq r_m \]

and

\[ x_j \geq 0 \]  
\[ j = 1, 2, 3, \ldots, n; \quad i = 1, 2, 3, \ldots, m \]

where

- \( Z \) = objective function to be maximized
- \( x_j \) = choice variables for which the problem is solved
- \( c_j \) = coefficient measuring the contribution of the \( j \)th choice variable to the objective function
- \( r_i \) = constraints or restrictions placed upon the problem
- \( a_{ij} \) = coefficient measuring the effect of the \( i \)th constraint on the \( j \)th choice variable

Equation (1) is the objective function. Equations (2), (3), (4) form the constraint set. These equations constrain the solution so that only available resources are used and/or desired production levels are achieved. The model solves for the \( x_j \). The final equation (5) is the non-negative constraint; this ensures that all choice variables are equal to or greater than zero. The equations are linear in the sense that the variable exponents are 1 and there are no variable cross products.
3. Case Study

The methodology of plantation management planning consists of four stages, i.e., data preparation, generation of management alternatives, selection of optimal management alternatives and management plan formulation (Figure 1).

![Diagram of methodology]

Figure 1. Schematic diagram of the methodology.
The management unit records will give information needed for evaluation of the yearly plan of operations. For this purpose the following information is needed:

3.1.1 Height, basal area, growing stock, growth, yield and site class estimated for each management unit from existing plot measurements.

3.1.2 Establishment. A detailed analysis of the past and proposed future management of establishment program.

3.1.3 Thinning. Thinning prescriptions are defined by the current annual increment remaining after thinning and by yield. The thinning interval and effects of thinning on growth and yield of trees are determined from plot measurements and tree ring analyses. These results are used in the simulation.

3.1.4 Operational Costs. The plantation cost records are examined for each activity. The records are translated to constant price terms using local price indices for labor, equipment, vehicles, materials and transportation. The future operational costs are reviewed and modified by manager and forester base on trends in this data wherever future operations are likely to result in different future costs. Forecasts for the silvicultural techniques are made solely on a budgeting basis.

3.1.5 Wood supply and demand. Schedules of wood supply from the plantation are obtained, both maximum and minimum quantities available to be purchased.

3.1.6 Management records and restrictions.

3.1.7 Other factors. Many other data will be collected covering such things as:

- Tax and discount rates,
- Survival expectations under different treatment,
- Reduction factors on fertilization, weed control and tree breeding responses, estimate from research data to reflect operational difficulties and scale, and
- A factor to represent loss in productivity equivalent to that expected from fire, insect and disease.

3.2 Generation of Management Alternatives

3.2.1 Forest Plantation Management Objectives

The ultimate objective is a harvest schedule that will meet imposed restrictions while producing a total present value that differs by a minimum amount from the value that would be obtained by implementing the optimum management regime in each cutting unit. In making a decision, the choice for the course of action to take is not always based upon a simple, single criterion like profit. Often, there are other important considerations. A decision should accomplish necessary and desired results to have significance; certain outputs are expected. At the same time, the resources that aid and limit the choice of alternatives must also be considered. These are the inputs available to implement the decision. The forest plantation management objectives can be classified as follows:

1) Determine the maximum sustained yield for a forested plantation area.
2) Define the structure of the forest plantation necessary to maintain the desired sustained yield.
3) Determine the optimal management regimes on plantation area.
3.2.2 Limiting Assumptions

The identification of alternative forest plantation management regimes rests on the management’s view of its objectives, environment, resources and constraints as a result of its assessment of the management situation. In general, it is vital for the management to have a good assessment of the controllable and non–controllable elements of the system. The controllable elements are the systems resources characterized by certain attributes. In the forest regulation system, these decision include length of the cutting cycle/rotation, timber stand treatments, length of planning periods/horizon, length of development/ conversion periods, road development schedules, and entry periods.

The non–controllable elements arise from limitations or constraints which management would have to work around. Such constraints may be brought about by biophysical, social and economic factors like topography, climate, structure of forest stands, species mix, site quality, and others that are beyond the control of the manager.

A management alternative is merely a set of specifications for the controllable elements subject to limitations imposed by non–controllable elements. In this study, a very large number of management alternatives can be avoided and can be specified for each management unit.

3.2.3 Identification of Forest Plantation Management Alternatives

After the limiting assumptions are specified, application of the system requires the definition of all permissible management regimes where a management regime is defined as a feasible pattern of successive harvests in a management unit during the planning period. Harvest yields and total present values under the various management regimes for a typical will be computed.

3.3 Selection of Optimal Management Alternative

A model that generates the management regimes provides an estimate of the contribution to corporate profit for each alternative regime considered for each management unit. The core of the system is the optimal management alternative. The optimal management alternative is formulated as a linear programming closely patterned after that by Ware and Clutter (1971); Dargavel and Bethune (1972); Johnson and Scheurman (1977); and Leuschner (1990). The activities considered in this formulation are the area of each management unit to be managed under each management regime. These activities are selected so that the total contribution to corporate profit is maximized.

3.3.1 Computation of Net Present Value (NPV).

To compute the NPV for forest plantation, the notation NPV\textsubscript{ijk} (net present value of management unit i under management regimes j) is used. This can be calculated by the formula:

\[
NPV_{ijk} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \frac{V_{ijk}}{(1+r)^{(k-1)}}
\]  

(10)

where,
NPV<sub>ijk</sub> = net present value per rai of management unit i (i = 1, 2, 3, ..., 11) under management regimes j (j = 1, 2, 3, ..., 14) at cutting period k

\[ V_{ijk} = \text{value per rai of management unit i if managed according to management regime j at cutting period k} \]

\[ r = \text{rate of interest, 5.75\%} \]

\[ y = \text{cutting period, 5 years} \]

\[ k = \text{cutting period i.e. } k = 1, 2, 3, \ldots, 6 \]

### 3.3.2 Management Constrains

The following management constraints are imposed:

1) **Area.** The management area must be not greater than the total area.

2) **Regulation.** The total of the standing volumes on each management unit at the end of the planning period for each selected management regime must exceed the quantity of growing stock required to sustain the yield after the end of the planning period, or the plantation must be regulated forest.

3) **Budget.** The cost of the forest operations (clearing, planting, fertilizing, thinning etc.) for each selected management regime must not exceed the budget available in each period.

### 3.3.3 The Linear Programming (LP) Model.

There are j management regimes, i management unit, with each regime producing a net present value. To mathematically represent the variables involved in the model the following notation is defined:

\[ i = \text{management unit (i = 1, 2, 3, \ldots, m)} \]

\[ j = \text{management regimes (j = 1, 2, 3, \ldots, n)} \]

\[ \text{NPV}_{ij} = \text{net present value of management unit i under management regimes j} \]

\[ M_{ij}R_j = \text{area of management unit i under management regimes j} \]

\[ B_{ij} = \text{budget of management unit i under management regimes j} \]

From the above notation, the cut-scheduling problem can be formulated. The objective function effort to maximize net present value (Z) of timber produce from the teak plantation during the planning horizon as required under FIO policy statutes and research database, where

**Objective function**

\[ \text{Max } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \text{NPV}_{ij} M_{ij}R_j \] \hspace{1cm} (11)

**Subject to**

Area constraint

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} M_{ij}R_j \leq \text{Total area of management unit i} \] \hspace{1cm} (12)
Budget constraint

$$\sum_{i=1}^{m} \sum_{j=1}^{n} MU_i MR_j \geq \text{Minimum budget/period} \quad (13)$$

Regulation constraint

$$\sum_{i=1}^{m} \sum_{j=1}^{n} MU_i MR_j \leq \text{Area/period} \quad (14)$$

Non–negative constraint

$$MU_i MR_j \geq 0 \quad (15)$$

Equation (11) is the objective function. Equations (12), (13), and (14) are the constraint set. These equations constrain the solution so that only available resources are used and/or desired production levels are achieved. The model solves for the $MU_i MR_j$. The final equation (15) is the non–negative constraint. This assures all choice variables are equal to or greater than zero. The equations are linear in the sense that the variable exponents are 1 and there are no variable cross products.

3.4 Solution Report

Through a linear programming model, the optimal solution to the forest plantation planning problem was derived using the LINDO 6.1 software. The report announces that an optimal solution has been found and summarized in Table 1. The value of the objective function at the optimum solution is 659,146,700 baht.

Table 1  Optimal management regimes for each management unit by using linear programming.

<table>
<thead>
<tr>
<th>Management unit</th>
<th>Optimal management regimes</th>
<th>Cutting area (rai)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8 (CP3; 1TP4; 2TP6)</td>
<td>210.35</td>
</tr>
<tr>
<td>2</td>
<td>5 (CP2; 1TP3; 2TP5)</td>
<td>532.10</td>
</tr>
<tr>
<td>3</td>
<td>5 (CP2; 1TP3; 2TP5)</td>
<td>84.31</td>
</tr>
<tr>
<td>4</td>
<td>8 (CP3; 1TP4; 2TP6)</td>
<td>247.77</td>
</tr>
<tr>
<td>5</td>
<td>4 (CP1; 1TP2; CP4;1TP5)</td>
<td>967.2</td>
</tr>
<tr>
<td>6</td>
<td>8 (CP3; 1TP4; 2TP6)</td>
<td>477.79</td>
</tr>
<tr>
<td>7</td>
<td>5 (CP2; 1TP3; 2TP5)</td>
<td>337.52</td>
</tr>
<tr>
<td>8</td>
<td>6 (CP2; 1TP3; 2TP5; CP6)</td>
<td>281.86</td>
</tr>
<tr>
<td>9</td>
<td>3 (CP1; 1TP2; 2TP4; CP5)</td>
<td>356.16</td>
</tr>
<tr>
<td>10</td>
<td>1 (CP1; 1TP2; 2TP4)</td>
<td>998.78</td>
</tr>
<tr>
<td></td>
<td>5 (CP2; 1TP3; 2TP5)</td>
<td>44.84</td>
</tr>
<tr>
<td></td>
<td>8 (CP3; 1TP4; 2TP6)</td>
<td>62.86</td>
</tr>
</tbody>
</table>
Table 1  Optimal management regimes for each management unit by using linear programming. (Continues)

<table>
<thead>
<tr>
<th>Management unit</th>
<th>Optimal management regimes</th>
<th>Cutting area (rai)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2 (CP₁; 1TP₂; 2TP₄; CP₆)</td>
<td>766.91</td>
</tr>
<tr>
<td></td>
<td>3 (CP₁; 1TP₂; 2TP₄; CP₅)</td>
<td>664.05</td>
</tr>
<tr>
<td></td>
<td>4 (CP₁; 1TP₂; CP₄; 1TP₅)</td>
<td>81.57</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>6,142.62</strong></td>
</tr>
</tbody>
</table>

Legend:

CPₜ  = Cut/harvesting, establishment and maintenance at cutting period t
1TPₜ  = First thinning at cutting period t
2TPₜ  = Second thinning at cutting period t

3.5 Management Plan Formulation

A Management Plan was formulated for the Thong Pha Phum forest plantation based on the optimal management alternatives identified from the LP. The Management Plan was displayed in the form tables and maps, which are convenient for implementing, monitoring and adjusting the Management Plan.

4. Conclusions and Recommendations

The management planning for forest plantation is a complex activity. Linear programming offers an optimal solution to the problem forest plantation management. The management alternatives were formulated as a linear programming model. The activities considered in this formulation are the area of each management unit to be managed under each management regime. The linear programming model is helpful in illustrating the economic interrelationships among rotation length, adopted period for conversion to sustained yield, and net present value of the forest. The LP model permits specification of the management program in an irregular trend that will maximize the net present value of expected yields while converting the existing forest or plantation to regulated forest in a finite period.

Finding the optimal solution to a linear programming model is important, but it is not the only information available. There is a tremendous amount of sensitivity information, or information about what happens when data values are changed. Sensitivity of the linear programming model to variations in the input data can be ascertained by systematically varying the factors that enter into the formulation of the optimization model. Information concerning the effects of these changes may be crucially important to the forest manager and can be used to develop solutions that exhibit stability in the presence of changes in certain poorly defined data inputs. A sensitivity analysis of the LP model is, therefore, important.

Successful implementation of the forest planning system will normally require repeated solutions at short time intervals because the basic input parameters are subject to frequent change. Since each forest ownership has its own particular objectives and problems, the system presented here will not be suitable for all forest plantations. However, the system is sufficiently well developed to illustrate the potential of
mathematical programming techniques for solving the regulation problem, and the techniques can be adapted without great difficulty for realistic application.

The management plan from this research is only a guideline for managing the forest. It should be transformed to an operational plan so that decision maker or manager could implement the operational plan conveniently. Some techniques namely PERT/CPM, network analysis could be used for the implementation of management plan.

References


