A heuristic for minimizing inventory and transportation costs of a multi–item inventory–routing system

Sombat Sindhuchao
Department of Industrial Engineering, Ubonratchathani University, 34000
somsin@rocketmail.com

Abstract

A heuristic for for minimizing inventory and transportation costs of a multi–item inventory–routing system is proposed in this paper. The system considered consists of a set of geographically dispersed suppliers and a central warehouse. Each supplier manufactures one or more non–identical items. The items are jointly replenished with an economic order quantity (EOQ) inventory policy and collected by a fleet of capacitated vehicles dispatched from the central warehouse with a frequency constraint. The warehouse stocks these unique items which face constant and deterministic demands from outside retailers. A constructive heuristic, a local search algorithm and a greedy randomized adaptive search procedure (GRASP) are developed for the inventory–routing problem. Solutions obtained from the GRASP are compared with the lower bound on the total costs obtained from a column generation approach. Computational experiments conducted on randomly generated problems show that the GRASP performs efficiently in finding good solutions.

Keywords: Inventory–routing; Multi-item; Transportation; Economic order quantity; Greedy randomized adaptive search procedure.

1. Introduction

Inventory control and transportation planning of an organization are traditionally managed by different departments each of which has its own goal. As a result, inventory and transportation costs are minimized separately by each department. Normally in the logistics system, there is a trade off between the inventory cost and the transportation cost. When attempting to decrease one cost, the other will normally increase. For instance, in an inbound material collection scenario using a simple EOQ inventory policy where a fixed ordering cost could be viewed as a fixed transportation cost, smaller order quantity leads to a lower inventory holding cost but a vehicle needs to be dispatched more frequently to collect materials which generates a higher transportation cost. However, the total inventory and transportation cost in the system can be greatly reduced if inventory control and transportation planning are closely coordinated.

In this research, an inbound commodity collection system is studied. The system consists of a central warehouse with unlimited space for stocking inventories and a set of geographically dispersed suppliers each of whom produces one or more but non–identical items that face constant and deterministic demands from outside retailers. The demand rates of items need not to be the same. The items are jointly replenished using economic order quantity (EOQ) policy. When the manager of the central warehouse decides to replenish the inventory, a private vehicle with limited capacity is sent to visit a set of dispersed suppliers to collect a group of items. No item can be assigned to more than one group and it is not necessary for items produced by the same supplier to be in the same
group. There are an unlimited number of vehicles available at the central warehouse. Nevertheless, in a given time horizon, the frequency of dispatching each vehicle is limited due to the time required for maintaining vehicles and for other responsibilities, and the limited material handling capacity. Upon the completion of product collection, the vehicle returns to the central warehouse where the products are unloaded and stored.

For each item, the decision variables, which are the replenishment quantity and the replenishment interval, must be determined along with an efficient route for each vehicle so as to minimize the total cost per unit time associated with this integrated inventory–transportation system.

There are a number of related papers involving combination of inventory and routing. In early 1980’s, Federgruen and Zipkin (1984) are the first to integrate the allocation and routing problem in a single model. They study the allocation of a scarce resource from a central depot to many retailers using a fleet of capacitated vehicles and consider random demands in a single period model. The problem is formulated as a non-linear integer program and interchange heuristics for the deterministic vehicle routing problem are modified to solve the problem. Later Anily and Federgruen (1990) consider a single-item distribution system with one depot and a set of retailers that keep inventories. They assume that the demand rate of each retailer is an integer multiple of some base rate. Anily (1994) extends the work of Anily and Federgruen (1990) by employing general holding cost rates. Another interesting research belongs to Viswanathan and Mathur (1997). They integrate a vehicle routing problem and inventory decisions in a single warehouse multi-retailer multi-product distribution system with deterministic demands. They propose a stationary nested joint replenishment policy (SNJRP) heuristic to solve the problem where replenishment intervals are limited to be power of two multiples of a base planning period. Qu, Bookbinder and Iyogun (1999) develop an integrated inventory and transportation system for joint replenishment with a modified periodic policy in which each replenishment period is an integer multiple of a base period. They propose a heuristic decomposition method to solve the problem.

In this paper, a greedy heuristic that can rapidly find a reasonably effective starting solution is proposed. A supplier exchange (SE) heuristic based on a 2–exchange heuristic is developed to search for an improved solution. A greedy randomized adaptive search procedure (GRASP) [see Resende (1998)] is also developed.

This paper is organized as follows. The model is formulated in section 2. In section 3, a greedy heuristic and a solution improvement heuristic and the GRASP are introduced. Computational experiments are discussed in section 4 and finally conclusions are provided in section 5.

2. Model Formulation

The system considered consists of a central warehouse and a set of geographically dispersed suppliers. The warehouse holds a set of items, S, manufactured by those suppliers and each item is unique. The demand rate $D_j$ for item $j$ and $j \in S$ is assumed to be constant and deterministic. Holding one unit of item $j$ costs $h_j$ per year where $h_j$ is constant. A fleet of $m$ identical vehicles with limited capacity $C$ are sent from the warehouse to collect items from suppliers. Each vehicle cannot travel more than a certain number of trips per year $F$. The total system costs are composed of the inventory holding cost, the fixed ordering cost, the fixed dispatching cost and the vehicle routing cost. For convenience, the fixed ordering cost and the fixed dispatching cost are combined in a single term $K$. To solve the inventory-routing problem is to determine the
subsets of items that are assigned to a single vehicle for item collection, the corresponding replenishment quantities, the replenishment interval and the optimal vehicle routes, that minimize the average total inventory and transportation cost per unit time.

It is assumed that the units of items are such that the corresponding quantities can be meaningfully added together. Let \( S \subseteq S \), and let \( Q_j \) denote the replenishment quantity of item \( j \). Then define

\[
Q(S) = \sum_{j \in S} Q_j \quad \text{and} \quad D(S) = \sum_{j \in S} Q_j
\]

to be the aggregate replenishment quantity and the aggregate demand for subset \( S \) respectively. The aggregate inventory holding costs with respect to the aggregate replenishment quantity can be defined as

\[
h(S) = \frac{\left( \sum_{j \in S} h_j D_j \right)}{D(S)}.
\]

The vehicle routing cost plus \( K \) is denoted by \( L(S) \). With both the vehicle capacity and frequency constraints, the aggregate replenishment quantity for the items in a feasible subset \( S \) can be determined from

\[
Q^*(S) = \max \left\{ \frac{D(S)}{F}, \min \left\{ \frac{2D(S)L(S)}{h(S)}, C \right\} \right\}
\]

and the corresponding optimal costs can be obtained from

\[
c(S) = \begin{cases} 
L(S)F + \frac{1}{2} h(S) \frac{D(S)}{F} & \text{if } \sqrt{\frac{2D(S)L(S)}{h(S)}} \leq \frac{D(S)}{F} \\
\sqrt{2D(S)L(S)h(S)} & \text{if } \frac{D(S)}{F} \leq \sqrt{\frac{2D(S)L(S)}{h(S)}} \leq C \\
L(S) \frac{D(S)}{C} + \frac{1}{2} h(S)C & \text{if } C \leq \sqrt{\frac{2D(S)L(S)}{h(S)}} 
\end{cases}
\]

The integrated inventory-transportation problem can now be formulated as a partitioning problem with the assumption of \( m \) vehicles.

\[
\min \sum_{i=1}^{m} c(S^{(i)})
\]

subject to \( \bigcup_{i=1}^{m} S^{(i)} = S \)

\( S^{(i)} \cap S^{(k)} = \emptyset \quad \text{for all } i, k = 1, 2, \ldots, m; \quad i \neq k. \)
3. Heuristics

The problem is to partition items into several groups or subsets in such a way that the average total integrated inventory and transportation cost per unit time is minimized. Clearly, exact solution of this problem is very hard to obtain. Therefore, heuristics are focused for this problem.

A greedy randomized adaptive search procedure (GRASP) is developed to solve this problem. GRASP has been applied to many difficult combinatorial optimization problems including the mixed Chinese postman problem [see Corberan et al. (2002)], the job shop scheduling problem [see Binato et al. (2002)] and the clustering problem [see Cano et al. (2002)]. GRASP is an iteration process that consists of two phases, a construction phase and a local search phase [Feo and Resende (1995)]. A feasible solution is generated in the construction phase and then improved with a local search heuristic in the local search phase. The best solution obtained is selected as the solution of the problem. A constructive heuristic called Distance Sum (DS) is used in the construction phase to generate an initial feasible solution. In the local search phase, a neighborhood search algorithm called Supplier Exchange (SE) is applied. A TSP tour of each vehicle is firstly constructed by using the Arbitrary Insertion heuristic [see Rosenkrantz et al. (1977)] and then improved by using the 2-opt exchange heuristic [see Croes (1958) and Lin (1965)]. All heuristics are described as follows.

3.1 Distance Sum (DS) Heuristic

In the DS heuristic, routes are constructed sequentially for one vehicle at a time. Assignment of items to a vehicle, one item at a time, is based on the idea that the best candidate should be an item whose supplier is located closest to the warehouse and also close to at least one supplier that is already in the route. Let \(d_{j_i j_2}\) denote the distance (cost) from the supplier of item \(j_1\) to the supplier of item \(j_2\), for all \(j_1, j_2 \in S\). Similarly, let \(d_{0 j}\) and \(d_{j_0}\) denote the distance from the warehouse to the supplier of item \(j\).

Step 0. Initialize an empty route for the next vehicle.
Step 1. For each of ungrouped items, say \(j\), that can be added to the vehicle without violating its capacity constraint, determine its distance–sum as the minimum value of \(d_{j'_i} + d_{0 j}\) over all items \(j'\) already assigned to the current vehicle. If the current vehicle does not contain any items, let the distance–sum be \(d_{0 j}\). If no such items exist, go to Step 3.
Step 2. Find the item with the smallest distance-sum, assign it to the vehicle, and return to Step 1.
Step 3. If all items have been assigned to a vehicle, go to Step 4. Otherwise, if there are available vehicles left, return to Step 0.
Step 4. Find a TSP tour for all vehicles using the Arbitrary Insertion heuristic to construct a route and the 2-opt exchange heuristic to improve the tour.
3.2 Supplier Exchange (SE)

In the SE approach, in each step, a group of items with a common supplier currently replenished by one vehicle are exchanged with a group of items with a common supplier that is currently replenished by another vehicle. The exchange occurs only when it is feasible and incurs cost reduction.

Step 0. Sequentially select the next vehicle to consider. If all vehicles have been considered without making any improving move, stop. Otherwise, return to the first vehicle.

Step 1. Sequentially select the next supplier in the current vehicle to consider for moving. If all suppliers have been considered, go to step 0.

Step 2. Consider only a feasible exchange. Determine the (approximate) cost changes of exchanging this supplier along with its items that are replenished with the current vehicle with any group of items with a common supplier currently replenished by another vehicle.

Step 3. Perform the exchange that results in the largest cost savings, if any, and go to Step1.

3.3 Greedy Randomized Adaptive Search Procedure (GRASP)

GRASP is an iteration process with two phases, a construction phase and a local search phase. In the construction phase, an initial feasible solution is generated and in the local search phase, the solution is improved with a local search method. The process is repeated for a predetermined number of times M and the best overall solution is considered as the solution of the problem.

In the construction phase, the DS heuristic is implemented to construct a route for each vehicle. However, for item selection in Step2 of the DS heuristic, the best item with smallest distance–sum is not chosen. Instead an item will be randomly selected from the restricted candidate list of items and assigned to the vehicle. Items in the list are the best item plus the unassigned items whose distance–sum is greater than the best distance–sum at most a certain value $\alpha$, say up to $\alpha = 40\%$ over the best value. To improve the solution obtained in the construction phase, the SE heuristic is applied in the local search phase and then the best solution is updated.

4. Computational Experiments

To test the performance of proposed heuristics, computational experiments have been conducted with four sets of generated random instances. The demand rate and the inventory holding cost rate for each item are randomly generated from the uniform distribution on $[100, 300]$ and $[1, 15]$ respectively. The items are randomly assigned to one of 10 suppliers. The locations of the warehouse and suppliers are generated uniformly in the square $[0, 20]^2 \subseteq \mathbb{R}^2$, and Euclidean distances are used to measure transportation costs, with unit cost per unit distance traveled. A base case has been defined as follows: Each vehicle has a capacity of $C = 150$ units with $F = 10$ trips per time unit. The fixed ordering and dispatching costs are set to $K = 50$. For the GRASP, the maximum number of runs M is set to 50. In addition, $\alpha$ is set to 60%. The size of an instance is identified by the number of items, $n$, and the number of vehicles, $m$. Six different problem sizes $(n=15, m=3), (n=30, m=6), (n=40, m=8), (n=50, m=10), (n=100, m=20)\text{ and } (n=200, m=40)$ are tested with ten instances for each problem size.
All the heuristics have been implemented in the C++ programming language on a PC with a 1.80 GHz Intel Pentium 4 CPU and 240 MB of RAM.

For the first four problem sizes, the quality of the solutions is assessed by comparing the heuristic costs to the column generation lower bound on the optimal costs computed in the work of Sindhuchao (2004). Figures 1–4 show the relative percentage of the error of the objective function values from DS, SE and GRASP with respect to the lower bound. Table 1 provides comparison of the computational time of all heuristics. For the larger problem sizes ($n=100$, $m=20$) and ($n=200$, $m=40$), the objective function values obtained from DS, SE and GRASP are compared to the best values among them. The results are given in Tables 2-3.

Table 1. Average computational time for the heuristics

<table>
<thead>
<tr>
<th>Problem Size</th>
<th>LB (second)</th>
<th>DS (second)</th>
<th>SE (second)</th>
<th>GRASP (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=15, m=3</td>
<td>13.5</td>
<td>0</td>
<td>0.003</td>
<td>0.0717</td>
</tr>
<tr>
<td>n=30, m=6</td>
<td>849.5</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.1282</td>
</tr>
<tr>
<td>n=40, m=8</td>
<td>14207.6</td>
<td>0.0016</td>
<td>0.0031</td>
<td>0.1734</td>
</tr>
<tr>
<td>n=50, m=10</td>
<td>48672.0</td>
<td>0.0060</td>
<td>0.0032</td>
<td>0.2251</td>
</tr>
</tbody>
</table>
From the computational results, the GRASP performs efficiently. In addition, the GRASP’s performance improves as the problem size increases. For all instances, the error bounds of the solutions obtained from the GRASP are about 3% on average. Moreover, the GRASP can find a solution very rapidly. For the large problem sizes \((n=100, m=20)\) and \((n=200, m=40)\), the GRASP outperforms the DS and SE heuristics. However, the gap between the solution from the GRASP and the ones from the other two heuristics seems to decrease as the problem size increases.

### 5. Conclusions

In this research, an inbound material-collection system with one warehouse, multiple supplies and multiple items is studied. The GRASP for an integrated inventory-routing problem in a deterministic setting is developed to find a near-optimal solution for the problem. The DS heuristic is utilized to find the solution in the construction phase and the SE local search method is introduced in the local search phase. The computational results show that the GRASP can find near-optimal solutions very rapidly with an average error bound about 3% for most cases. The solution quality seems to increase as the problem size increases. For the future research, more efficient local search methods applied in the local search phase of the GRASP will be investigated.

### References


